Estimation of vertical subgrade reaction coefficient from CPT investigations: applications in Christchurch

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ABSTRACT

The vertical subgrade reaction coefficient Kv is extensively used by structural engineers for the design of shallow foundations. The value of the coefficient is usually estimated by the geotechnical engineer by three possible ways: in situ testing, laboratory testing and bibliography. In this paper a methodology is proposed for estimating the subgrade reaction coefficient K_{CPT} from cone penetration test. The continuous nature of CPT results provides a detailed stratigraphy that can be used for delineating the coefficient with depth which could be invaluable for both structural and geotechnical engineers. Comparison of results from one CPT with the corrected (N_{60}) SPT values from an adjacent borehole constructed on a site in Christchurch indicates that the results of the proposed method may be of some use. The collection of further results from site investigations is recommended for verifying the validity of the proposed method in sands of any state of packing.

1 INTRODUCTION

The coefficient of subgrade reaction k is a conceptual relationship, which is defined as the soil pressure σ exerted divided by the measured deflection δ:

$$k=\sigma/\delta \qquad (1)$$

The parameter k is widely used in the structural analysis of foundation members (shallow and deep foundations) and is measured in kN/m^3. The usual procedure for measuring k is to perform in situ plate load tests. The test is fully described in BS EN 1997-1:2004 (BSI, 2004), ASTM D 1194-94, Tomlinson (1980), BS 1377:1990 (Part 9), Clayton et al. (1995). The usual practice is to load small steel plates, 30-75cm diameter and 25.4mm thickness, which are embedded in shallow pits (or boreholes, Wrench, 1984) and record the measured deflection by means of dial gauges to the nearest 0.025mm. The test can be performed either with constant rate of penetration or by applying incremental loads (Simons et al., 2002). The plate load test for a number of reasons has the following limitations (Barounis et al., 2007 and 2011):

- Size effect. The results of the test reflect the settlement characteristics of the soil within the pressure bulb of the plate. The pressure bulb of the actual foundation is much deeper as than that of the plate.
- Scale effect. The ultimate bearing capacity of saturated clays is independent of the plate size. For cohesionless soils it increases with plate size. In order to reduce this effect, two or three tests should be performed with different plate sizes and the bearing capacity of the actual foundation extrapolated.

- Time effect. The test is of short duration and as a result the load-settlement curve obtained is not always representative.
- Reaction loads. Since the applied loads can be greater than 300kN, a truck or some other form of kentledge is necessary.
- Experienced technicians should always be involved.
- High cost per test and sufficient number of tests can result in a substantial increase in the site investigation cost.
- Water table effect: The level of the water table can affect the bearing capacity of sandy soils. If the water is above the footing level, the water has to be pumped before the plate is placed. This can cause extra delays and cost increase.

Therefore, a methodology is proposed whereby k is measured in the field by means of the CPT. The proposed methodology can be applied mainly for shallow foundation design but it may also be extended for deep foundations and liquefaction analysis.

2 PROPOSED METHODOLOGY FOR CALCULATION OF K_{CPT}

The proposed method is to obtain k directly from CPT's by dividing the stress q_c applied on the ground by the amount of deflection δ, taken to be equal to the cone penetration into the ground. Usually the amount of deflection δ that corresponds to a value of q_c is the same increment for the whole depth of the test and ranges between 25 and 50 millimeters. Every pair of values of q_c and δ that are recorded during testing may be used until the final depth is reached. The Opus CPT rig used for this paper records q_c for every centimeter of penetration, thus $K_{CPT}=q_c/0.01$ and when σ is in MPa, then k is in MN/m^3. The k values should be computed for every depth increment until the final depth of the test enabling a continuous plot of k versus depth to be prepared. From this graph the modulus of subgrade reaction can be delineated with depth which enables the direct calculation of the subgrade reaction modulus for foundations of any shape, at any depth of interest by applying well established methodologies (Bowles, 1997; Das, 1990). The transformation from K_{CPT} to $K_{FOUNDATION}$ can be performed by applying formulae presented in the next paragraphs of this paper.

The advantages of measuring k in the field by CPT testing are:

- Great advantages of CPT testing over plate load testing include: less cost; less time; no requirement for kentledge or reaction to produce high stresses on hard soils; no need for dewatering of excavations to perform plate loading; independence from plate load size effects.
- It is impossible for a plate load test to impose on the ground similar levels of stress as in a CPT test, which in the latter case, can be as high as 40 Mpa.
- In situ testing can be performed on any type of soils (fine grained, coarse grained, sensitive, organic, peats) without the need for excavation.
- Testing at great depths can be performed, which is in contrast with plate load testing. For such depths plate loading can only be performed from within a borehole which is a very expensive option.
- Measured K_{CPT} can be calculated for any depth and can be transformed to $K_{FOUNDATION}$ for any shape of shallow foundation or pile. The proposed analysis can be extended to depths where induced foundation stress is reduced to 20%. The final design value for $K_{FOUNDATION}$ may be computed for each discrete layer detected from CPT testing and by averaging or using other mathematical techniques; a reliable value can be estimated.
- Low cost compared to plate load testing.
- During cone penetration, the soil is taken to failure and the response of K_{CPT} at failure is recorded. This is difficult to achieve in-situ with plate load testing and sometimes unnecessary.
- Provides another tool for assessing the soil as a foundation material under static conditions and may also prove useful for assessing the liquefaction potential of saturated sands and silts. The measured K_{CPT} values for cohesive and cohesionless soil types can be correlated with the consistency of clays and relative density of sands or SPT blows N or N_{60}, and soil behaviour.

- The proposed analysis can be extended to any desired depth for performing foundation analysis of any foundation shape.

The limitations compared to plate, consolidation and unconfined compression tests are:

- The loading conditions during cone penetration do not reflect the in situ loading from foundations or plate load tests, i.e. the CPT measures properties at failure, whereas in situ loading or foundations perform within the elastic range. Thus the calculated values of the subgrade reaction coefficient from the proposed method are considerably higher than the values measured with the plate load and can be considered as an upper bound.
- Limited capacity to penetrate through hard soils or refuse on gravels may result in termination of the test with insufficient K_{CPT} values.

The cone penetration test (CPT) is now widely used for geotechnical site characterisation including liquefaction potential assessment and in-situ determination of soil properties (Salgado, 2008). The test is performed by pushing a 35.7 mm diameter penetrometer vertically into the ground at a penetration rate of 20mm/s. The tip resistance of the cone q_c, defined as the vertical force in MPa acting on the penetrometer tip divided by the base area of 1000mm^2, is recorded during the test versus testing depth. The penetrometer can also measure sleeve friction f_s, defined as the shear force applied on the cylindrical sleeve located above the tip divided by the 15.000 mm^2 standard sleeve area. Pore pressures u can also be measured during penetration in the piezocone version of CPT testing and in this case the total cone resistance q_t is calculated by:

$$q_t= q_c + (1-a)\, u \qquad (2)$$

where
q_t=corrected cone resistance [MPa]
q_c= tip cone resistance [MPa] measured at any depth
a=dimensionless area ratio= 0.70 to 0.85
u=measured pore pressure [MPa] at the same depth as q_c

Modern CPT rigs record q_c, f_s and u for every centimetre of cone penetration in to the ground allowing q_t to be calculated using equation 2. The soil types through which the cone penetrates can be determined based on the soil behaviour during penetration, as expressed in the combination of q_c, f_s and friction ratio defined as f_s/q_c.
During cone penetration the soil is initially compressed and then sheared to failure at a stress equal to the measured value of q_c. The rate of cone penetration of δ=2 centimetres per second classifies this type of testing as strain controlled in-situ testing, similarly to the laboratory triaxial and unconfined compression tests. Each soil layer can be perceived as a series of vertical springs 1 cm long, each spring having an ultimate reaction coefficient value of K_{CPT}, which is fully mobilised during cone penetration. The cone penetration test offers the ability to measure this ultimate reaction coefficient K_{CPT} for the complete depth of the profile as both q_c and δ are measured.
The CPT may be used for the calculation of the subgrade reaction coefficient K_{CPT} by using the proposed equation:

$$K_{CPT}= q_c/\delta=q_c/(0.01m)= 100\, q_c \qquad (3)$$

where
K_{CPT}= coefficient of subgrade reaction from CPT testing in MN/m^3
q_c= tip cone resistance in MPa measured at any depth
δ=cone penetration of 1 centimeter
Values for K_{CPT} may be calculated by using q_t instead of q_c in equation 1, when testing clay layers with high positive excess pore pressures. It is recommended that q_t values be used for the calculation of K_{CPT} whenever excess pore pressure measurements are prevailing, which is the case for cohesive soils.
In this paper results of K_{CPT} on sandy soils only are presented. In the case of sandy soils q_t=q_c (Robertson and Cabal, 2010).

3 PROCEDURE OF ANALYSIS FOR CALCULATION OF K_{CPT} VALUES

The proposed procedure for calculating the k values from cone penetration tests is simple: the K_{CPT} values are obtained by dividing the applied tip cone resistance q_c in MPa by the amount of penetration δ which is 1 centimetre for the Opus CPT rig used for this paper (refer equation 3). The values of K_{CPT} are considerably higher than the k values measured from plate load tests ($K_{0.3}$) since those k values generally tend to decrease as the foundation width B increases (Terzaghi, 1943). A typical foundation can be between 30 to 600 times larger than the 35.7 mm cone penetrometer. This means that the $K_{0.3}$ value obtained from an in situ test on a 0.3 m diameter plate is smaller than the K_{CPT} value and the value required for a foundation, $K_{FOUNDATION}$, is even smaller than the $K_{0.3}$ value due to the foundation breadth being greater than 0.3 m. Thus the following relation holds: $K_{CPT}>>K_{0.3}>K_{FOUNDATION}$.
The following mathematical transformation mechanism is proposed for converting K_{CPT} to $K_{0.3}$ reference value that can be useful to the structural designer. A reference value for the coefficient of subgrade reaction $K_{0.3}$ was adopted based on plate load testing with a plate width of 30 centimetres on the same soil and depth.
For transforming K_{CPT} values to $K_{0.3}$ reference values the following equation is proposed:

$$K_{0.3}=K_{CPT} \text{ x } \frac{D_{CPT}}{30} \tag{4}$$

where D_{CPT} is the cone diameter used and 30 is the plate diameter in cm.
The usual cone penetrometer diameter D_{CPT} is 3.57cm hence $K_{0.3}$ is given by:

$$K_{0.3} = K_{CPT} \text{ x } (3.57/30) =0.119\ K_{CPT} \tag{5}$$

4 COMPARISON OF $K_{0.3}$ FROM CPT AND SPT TESTING

For comparing the $K_{0.3}$ values obtained from the proposed method and some well established methods that are based on SPT blow counts, some field testing was undertaken in Christchurch. The field testing was limited in one borehole and one CPT, each one to 10 m depth. The distance between the two investigations was 9 meters. The depth of 10 m is considered sufficient for a typical shallow foundation design because the usual foundation depths are considerably less than this depth. The SPT N values were obtained at the same depth as the CPT was carried out with 1.5 m spacing of SPT testing.
A standard and calibrated electronic cone with a 60 degree apex angle with a diameter of 35.7 mm and a cross sectional area of 10 cm^2 was used for the collection of field data. Recording of cone resistance, deflection, pore pressure and sleeve friction was undertaken for every centimetre of cone penetration.
The standard penetration test (SPT) was performed in accordance with ASTM D 1586 (ASTM, 2000). A split spoon sampler was used with the retained sample described according the NZ Geotechnical Society's 'Guidelines for Field Classification and Description of Soil and Rock for Engineering Purposes' (2005). The number of blow counts N for penetrating 300 mm into the soil was measured. A calibrated hammer was used with an energy efficiency of 62%. The obtained SPT blows were corrected to N_{60} values according to the procedure proposed in the Appendix C3 of the Guidance on Repairing and Rebuilding Houses by the Canterbury Earthquakes (MBIE, 2012).
The tip cone resistance q_c measured has been used for calculating the coefficient K_{CPT} for the total depth of 10 m by using the proposed methodology. These values were then transformed to $K_{0.3}$ values according equation 5.
The corrected SPT blows N_{60} were also transformed to $K_{0.3}$ values by using the relationship proposed by Scott (1981) applicable to coarse grained soils:

$$K_{0.3}= 1.8\ N \tag{6}$$

where $K_{0.3}$=coefficient of subgrade reaction for a 30 cm diameter load plate [MN/m^3]

N= corrected SPT blow N

For reasons of comparison a second relationship was also used (Moayed and Janbaz, 2011) applicable to dense to very dense gravelly soils:

$K_{0.3}$= 2.821 N (7)

where $K_{0.3}$=coefficient of subgrade reaction for a 30 cm diameter load plate [MN/m^3]
N= corrected SPT blow N

The results from all three methods applied are shown in Table 1. The groundwater table during CPT and SPT testing was constantly at 1 m below ground level.

Table 1: Results of vertical reaction coefficient K_{CPT} and $K_{0.3}$ from SPT and CPT data

Depth of tested layer (from-to, in meters) and soil description	Range of q_c (MPa)	Range of K_{CPT} (MN/m^3)	Range of $K_{0.3}$ from CPT testing (eqn. 5) [average] (MN/m^3)	$K_{0.3}$ (eqn. 6) (MN/m^3)	$K_{0.3}$ (eqn. 7) (MN/m^3)	Corrected SPT N_{60}
1.65 to 1.95 Silty fine Sand	0.89 to 2.28	89 to 228	10.6 to 27.1 [21.4]	10.8	16.9	6
3.15 to 3.45 Silty fine Sand	2.12 to 3.85	212 to 385	25.2 to 45.8 [30.8]	8.1	12.7	4.5
4.65 to 4.95 Silt with some sand	2.81 to 5.26	281 to 526	33.4 to 62.6 [47.6]	19.8	31	11
6.15 to 6.45 Silt with some sand	0.96 to 1.79	96 to 179	11.4 to 21.3 [14.8]	17.1	26.8	9.5
7.65 to 7.95 Fine to medium Sand	0.74 to 10.45	74 to 1045	8.8 to 124.4 [61.8]	32.4	50.8	18
9.15 to 9.45 Fine to medium Sand	3.9 to 11.18	390 to 1118	46.4 to 133 [88.7]	32.4	50.8	18

5 DISCUSSION OF RESULTS

The relationship proposed by Scott has been extensively used in foundation design in many countries for more than thirty years. The recorded performance and behaviour of such foundations during this time span has been within acceptable limits. By comparing the average values obtained from the proposed methodology with the values from Scott's relationship, is evident that the proposed method generally produces higher values. The results of the comparison are shown in Table 2. The proposed method overestimated the subgrade reaction coefficient in the majority of the measurements between 98% and 280%.

The results from the proposed method are also compared with the results from Moayed and Janbaz method. In this case the proposed method overestimated the subgrade reaction coefficient in the majority of the measurements between 22% and 143%

The results indicate that the proposed method produces higher values for the subgrade reaction coefficient. The authors believe that the main reason for that are the differences between CPT and SPT testing. In CPT testing the soil is forced to failure by the pushing action of the cone that forces the soil downwards and sideways. In contrast, during SPT testing, the soil is not at failure and only shear stresses are developed between the external and internal sides of the split spoon sampler with compressive stresses to be developed under the cutting shoe of the sampler. The developed shear stresses and compressive stresses are not considerably high to produce the failure of the tested soil, which this is the case for CPT testing.

A second main reason for the overestimation of the coefficient is the continuous recording of the CPT with depth. In contrast, the SPT can be used only at certain depths, 1 to 1.5 meters apart. This is also considered as the main advantage of the proposed methodology compared to the SPT methodology. The geotechnical designer can then apply statistical techniques before adopting the final design values for the specific foundations.

The values obtained from SPT and plate loading testing is considered to load the tested soils within their elastic range. The plate load test is usually terminated when the tested soil exhibits 25 mm of measured settlement. Thus the soils tested by these two tests do not fail during testing. In this case the factor of safety against bearing capacity failure is always greater than 1. During CPT testing the factor of safety is always 1 at any depth and this is considered the main reason for the produced higher coefficient values. From the analysed data, the obtained values from CPT testing may be up to 2.8 times higher than the ones produced from the SPT analysis. This is considered to be consistent with the difference in the nature of the two tests.

Table 2: Results of vertical reaction coefficient K_{CPT} and $K_{0.3}$ from SPT and CPT data

Depth of tested layer (from-to, in meters) and soil description	$(K_{CPT}-K_{SCOTT})/K_{SCOTT}$	$(K_{CPT}-K_{M})/K_{M}$
1.65 to 1.95 Silty fine Sand	98%	27%
3.15 to 3.45 Silty fine Sand	280%	143%
4.65 to 4.95 Silt with some sand	140%	54%
6.15 to 6.45 Silt with some sand	-13%	-45%
7.65 to 7.95 Fine to medium Sand	91%	22%
9.15 to 9.45 Fine to medium Sand	174%	75%
$K_{SCOTT}= K_{0.3}$ subgrade reaction coefficient from eqn.6 $K_m= K_{0.3}$ subgrade reaction coefficient from eqn.7		

6 CALCULATION OF $K_{FOUNDATION}$ FOR A FULL SIZE FOUNDATION FROM K_{CPT}

By using the coefficient values produced from the proposed methodology, the final subgrade reaction coefficient of the actual foundation may be estimated by using existing published relationships.

The usual types of shallow foundations that can be analysed by the proposed method are:

- strip or long footings with $L/B \geq 5$
- rectangular pad foundations with dimensions L x B
- raft (or mat) foundations with dimensions L x B

where L=length and B=width of the foundation.

For a strip, pad or raft foundation, with dimensions BxL founded on medium dense sand or stiff clay, the K_{BxL} value can be calculated as follows:

$$K_{BxL}=K_{0.3} \times \left(\frac{m+0.5}{1.5m}\right) = 0.119\, K_{CPT} \times \left(\frac{m+0.5}{1.5m}\right) \quad (8)$$

where m = L/B (Bowles,1997).

For the same foundations on sands of any relative density, the K_{BxL} value can be calculated as follows (Bowles, 1997):

$$Ks= K_{0.3} \left(\frac{B+B_1}{2B}\right)^2 = 0.119\, K_{CPT} \left(\frac{B+B_1}{2B}\right)^2 \quad (9)$$

where B_1=0.3 m (reference plate width) and B=actual foundation width.

7 CONCLUSIONS

A methodology for estimating the coefficient of subgrade reaction from CPT testing is introduced in this paper applicable to sandy soils. The methodology was tested on a soft site in Christchurch and returned higher values for the subgrade reaction coefficient than the ones calculated from SPT testing. A factor of safety of 3 is strongly recommended to be applied on the calculated K_{CPT} values on the results from this method that someone performs before these results are used for final structural design. As only one borehole and one CPT were analysed, is strongly recommended that further site data should be analysed in the future for verifying the proposed methodology. For the time being the analysis presented is limited to sandy soils with measured in situ SPT N values between 6 and 19 blows. Thus the method needs still to be proved that gives consistent values for the complete set of blows N between 1 and 50.

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