Effect of soil profile modulus distribution on pile head lateral stiffness

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ABSTRACT

The effect of pile shaft diameter and soil Young's modulus distribution with depth on unrestrained pile head lateral and rotational stiffnesses of long piles is considered. Analysis of field test data for lateral load tests on piles of different diameters at particular sites had lead to the suggestion that modulus of subgrade reaction appears to increase with increasing pile shaft diameter. This is contrary to the usual understanding that the modulus of subgrade reaction is independent of pile shaft diameter. This puzzle is resolved in the realisation that only if the soil modulus is constant with depth is a constant modulus of subgrade reaction appropriate. A corollary of the work is that accurate estimates of pile head stiffness require better than routine site investigation data. Included in the paper is discussion of the effect of the variation with pile shaft diameter of the unrestrained pile head lateral and rotational stiffnesses for three distributions of soil modulus with depth, as well as the effect of the pile head moment to shear ratio.

1 INTRODUCTION

When pile foundations are to be constructed it is not uncommon to construct a test pile(s) to evaluate the capacity and also the stiffness. Sometimes these piles are at reduced scale and in others they are at prototype scale. There are in the literature only a few case studies in which the lateral pile head stiffness is measured on piles of different diameters at one site. The classic paper of Terzaghi (1955) considered lateral pile stiffness from the point of view of the Winkler spring model of pile-soil interaction (constant stiffness soil-pile springs). The conclusion is that as the diameter of the pile increases the coefficient of subgrade reaction (units FL^{-3}) for the soil decreases linearly with pile shaft diameter and consequently the modulus of subgrade reaction (units FL^{-2}) is independent of pile diameter. As will be seen below an important part of this argument is that the ground in which the pile is embedded has constant properties with depth.

The writer has been aware of an apparent effect of pile shaft diameter on lateral stiffness of piles since the thesis of Carter (1984) in which published data from lateral load testing of piles of differing diameters at particular sites were examined. Such data are sparse, but the few case histories available led to the finding that the modulus of subgrade reaction appears to increase with increasing pile diameter. The results of the evaluation of the initial lateral stiffness of free head piles are plotted in Fig. 1. Most of the data from which the points in these diagrams were derived are from back analysis of the field test data of Gill (1968), Gill and Demars (1970), Reese et al (1974), Reese and Welch (1975), and Dunavant and O'Neil (1985). The left part of Fig. 1 shows that the constant modulus of subgrade reaction idealization does not give the correct modeling for different diameter piles. A similar observation is also made by Reese and Van Impe (2001). The data in Fig. 1a were calculated on the assumption of a constant modulus of subgrade reaction. The failure of this idealisation to model the observed change in pile stiffness with change in pile diameter presents something of a quandary as simple dimensional analysis indicates that the

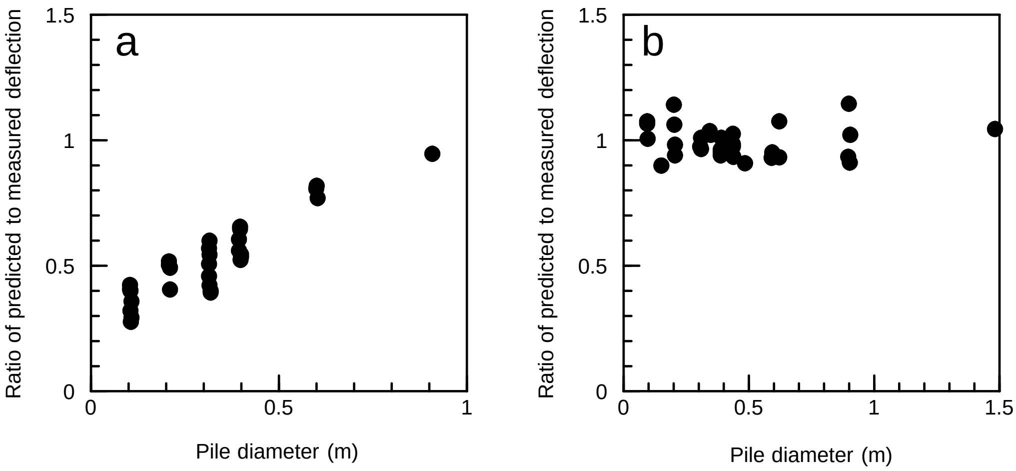

Figure 1: Evidence of the apparent size effect on pile lateral stiffness: (a) Unsatisfactory modelling when the modulus of subgrade reaction is independent of pile diameter, (b) satisfactory modelling when the modulus of subgrade reaction increases linearly with pile diameter.

Terzaghi conclusion must be sound. Fig. 1b presents satisfactory modelling of the lateral stiffness of different diameter piles. This is achieved by increasing the modulus of subgrade reaction as the pile diameter increases. Several possible explanations were investigated for this puzzling diameter effect, including three-dimensional finite element studies to investigate the action of shear stresses mobilised at the interface between the pile shaft and surrounding soil, Satyawan (2000). None of these indicated any size effect.

However, eventually settle a simple explanation was found. Namely that if the soil modulus increases with depth, larger diameter piles will have a greater lateral stiffness than one would expect from extrapolation of lateral load test data on smaller diameter piles and vice versa. It is the purpose of this paper to explain this in more detail. The main vehicle for our explanation is a group of expressions for the components of the pile head stiffness matrix given by Gazetas (1991), which are based on numerical calculation of the response of an elastic pile embedded in an elastic soil.

2 PILE HEAD STIFFNESS MATRIX FOR LONG PILES

In considering long piles the active length concept is useful, Gazetas (1991). This is the length of pile shaft beyond which deflections and rotations induced by head loading are negligible. Intuitively one can to think of this as the maximum depth which the pile "reaches" into the soil profile to mobilise lateral resistance. The active length depends on the diameter of the pile shaft, the Young's modulus of the pile shaft relative to that of the soil in which it is embedded, and the profile of the soil Young's modulus with depth. Gazetas gives expressions for the active lengths and the components of the pile head stiffness matrix in soil profiles having a constant modulus, a modulus increasing from zero at the ground surface as the square root of the depth, and a modulus increasing linearly with depth from zero at the ground surface. The definition diagrams for these three cases are given in Fig. 2 and Table 1 gives expressions for the active lengths and components of the pile head stiffness matrix. The active length equations in Table 1 are for dynamic lateral loading, equations are given elsewhere for the active lengths under static loading, the values obtained are similar to those for dynamic loading.

The pile head stiffness matrix relates lateral and rotational displacements of the pile head induced by shear forces and moments applied to the pile head as follows:

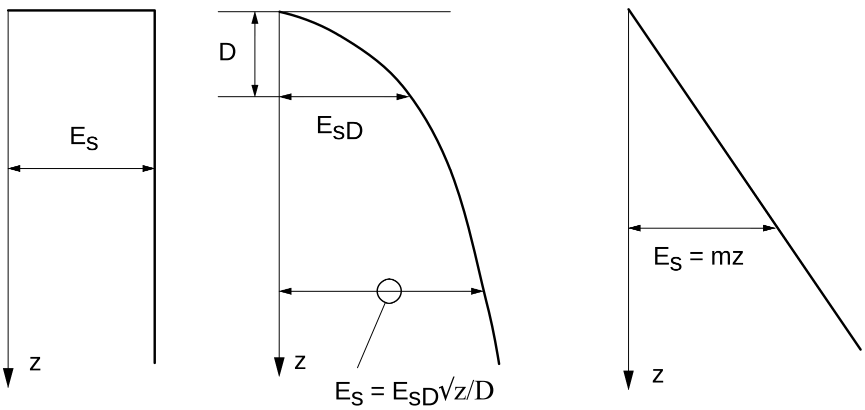

Figure 2: The three soil profile stiffness models used herein. (D pile shaft diameter, E_{sD} soil Young's modulus at a depth of one pile diameter.)

Table 1: Components of the pile head stiffness matrix for the soil profiles shown in Fig. 1

Model	K	L_a	K_{HH}	K_{HM}	K_{MM}
Constant	$K = \frac{E_p}{E_s}$	$L_a = 2DK^{0.36}$	$K_{HH} = DE_s K^{0.21}$	$K_{HM} = -0.22D^2 E_s K^{0.50}$	$K_{MM} = 0.10D^3 E_s K^{0.75}$
Linear	$K = \frac{E_p}{mD}$	$L_a = 2DK^{0.20}$	$K_{HH} = 0.60DE_{sD} K^{0.35}$	$K_{HM} = -0.17D^2 E_{sD} K^{0.60}$	$K_{MM} = 0.15D^3 E_{sD} K^{0.80}$
Parabolic	$K = \frac{E_p}{E_{sD}}$	$L_a = 2DK^{0.22}$	$K_{HH} = 0.80DE_{sD} K^{0.28}$	$K_{HM} = -0.24D^2 E_{sD} K^{0.53}$	$K_{MM} = 0.15D^3 E_{sD} K^{0.77}$

$$\begin{Bmatrix} H \\ M \end{Bmatrix} = \begin{bmatrix} K_{HH} & K_{HM} \\ K_{MH} & K_{MM} \end{bmatrix} \begin{Bmatrix} u \\ \theta \end{Bmatrix} \quad (1)$$

where: H and M are the shear force and moment applied to the pile head,
u and θ are the lateral displacement and rotation of the pile head,
and $K_{HM} = K_{MH}$.

As field testing is usually done on piles with unrestrained heads, the unrestrained pile head lateral and rotational stiffnesses are given by:

$$K_h = \frac{K_{HH} K_{MM} - K_{HM}^2}{K_{MM} - eK_{HM}} \quad (2)$$

$$K_\theta = \frac{K_{HH} K_{MM} - K_{HM}^2}{K_{HH} - K_{HM}/e} \quad (3)$$

where: K_h is the unrestrained horizontal stiffness of a free head pile,
K_θ is the unrestrained rotational stiffness of a free head pile,
e is the ratio M/H.

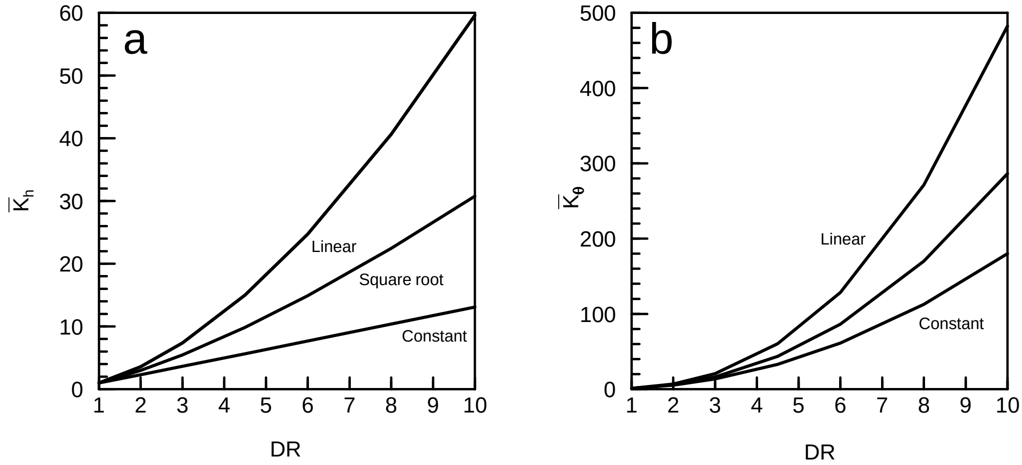

Figure 3: Pile head stiffness ratio as a function of pile shaft diameter ratio for M/H ratio = 0.1. (The starting value for the curves is 1.0.) (Shear wave velocity of the constant modulus soil profile 500 m/sec.)

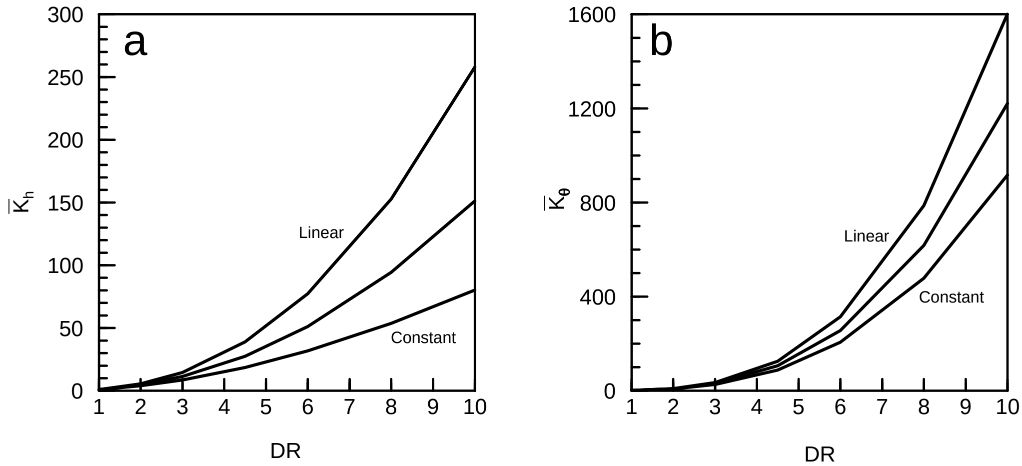

Figure 4: Pile head stiffness ratio as a function of pile shaft diameter ratio for M/H ratio = 10. (The starting value for the curves is 1.0.) (Shear wave velocity of the constant modulus soil profile 500 m/sec.)

3 EXPLANATION

Using equations 2 and 3 the effect of pile diameter on pile head stiffness for the various distributions of soil Young's modulus with depth can be investigated. Pile diameters between 0.2 m and 2.0 m were considered and the Young's modulus was set at a value for a reinforced concrete pile. Two limiting cases for the soil profile stiffness, shear wave velocities of 500 and 75 m/sec, were used. As only the initial parts of the load - displacement and moment - rotation curves are discussed herein, the shear wave velocity is used to charactise the soil as this gives the soil stiffness that controls the linear portion at the beginning of the pile lateral response. Assuming a value of 0.5 for the Poisson's ratio of the soil, the Young's modulus can be obtained from the shear wave velocity. For the linear and parabolic modulus distributions this Young's modulus is for a depth of one pile diameter. With the varying pile diameters the parameters were adjusted so that the modulus distribution with depth was the same for all pile diameters. It is apparent from equations 2 and 3 that the pile head stiffnesses depend on the ratio of the shear to moment applied at the pile head. We will consider the results with M/H = 0.1 and M/H = 10.0.

The unrestrained pile head lateral and rotational stiffnesses are plotted as ratios of the stiffness at a given diameter pile divided by the corresponding stiffness for the smallest diameter pile. The stiffness ratios are denoted by $\overline{K}_h$ and $\overline{K}_\theta$ and the ratio of the diameter of the current pile to the diameter of the smallest pile as DR. In Fig. 3 for data are for the case where M/H = 0.1, and in Fig. 4 the same stiffness ratios are plotted for the case where M/H = 10. Since stiffness ratios are plotted in Figures 3 and 4 the effect of the shear wave velocity, which is reflected in much larger stiffness values for the piles in the 500 m/sec soil profile, is not apparent as the ratios are very nearly the same for shear wave velocities of 75 and 500 m/sec.

Figs. 3 and 4 show that the unrestrained pile head rotational stiffness is more sensitive to pile diameter than the lateral stiffness; a consequence of the K_{MM} terms in Table 1 being a function of D^3, whereas the K_{HH} terms depend on D.

Figures 3 and 4 provide the explanation for the apparent pile size effect. The figures show that as the pile shaft diameter increases the unrestrained pile head stiffnesses increase for all three distributions of soil modulus. However, the stiffnesses for the linear and square root profiles increase at a faster rate than those for the constant modulus profile. The modelling on which Fig. 1b is based is equivalent to using a linear soil modulus distribution with depth. Although the calculations were based on a constant value for the modulus of subgrade reaction, the increase in modulus with increasing pile diameter means, in effect, that a value from deeper in the soil profile is used as a "representative" value as the diameter increases. Thus the "puzzle" in Fig. 1 is resolved in the realisation that it was not appropriate to model the soil profiles in which the piles were embedded as having a constant modulus of subgrade reaction with depth. As explained above the active lengths specified in Table 1 give an indication of how far into the soil profile the pile "reaches" to mobilise lateral stiffness. As the active lengths increase with increasing pile shaft diameter, the depth of an equivalent constant modulus also increases.

The lesson from this is that if the lateral stiffness of piles is required, more than a cursorily routine site investigation will be needed to estimate the pile stiffness adequately.

One very well documented and executed field test case history, which supports the need for good rather routine soil profile data, is reported by Ashford and Juirnarongrit (2003). At this site shear wave velocities indicate an upper layer about 6 metres deep, with an approximately constant shear wave velocity of 315 m/sec., below which the velocity increases (interestingly SPT data do not give such a clear picture). Reinforced concrete piles 0.4 m to 1.2 m in diameter were constructed and subject to dynamic lateral loading. From the values for the natural periods of these piles the Young's modulus of the soil profile was estimated. The results indicated that the change in stiffness with pile diameter could be modelled by assuming a constant soil modulus with depth. This result is consistent with the calculation of the active lengths for the various diameter piles, using equation 2 for the constant modulus soil profile and a pile Young's modulus of 25 GPa, all of which fall within the upper constant modulus part of the soil profile.

Further discussion of the above effects is given by Pender et al (2007).

4 CONCLUSIONS

The following conclusions are reached:

(i) The apparent pile size effect on unrestrained pile head lateral stiffness, highlighted by Carter (1984), Pender (1993) and investigated further by Pranjoto (2000), is a consequence of the distribution of soil modulus with depth. If the soil modulus increases with depth, then calculations based on a constant modulus distribution will underestimate the increase in lateral stiffness as the pile shaft diameter is increased. This conclusion is based on Figs. 3a and 4a.

(ii) Figs. 3b and 4b show how the unrestrained pile head rotational stiffness increases with pile shaft diameter. As for the lateral stiffnesses, the values of the rotational stiffnesses for the square root and linear modulus distributions increase faster than those for the constant modulus profile.

(iii) A corollary of (i) and (ii) is that good quality soil profile data is imperative for making reliable estimates of the initial pile head lateral and rotational stiffnesses. The work of Ashford and Juirnarongrit (2001) shows that a shear wave velocity profile gives a better indication of soil stiffness properties than Standard Penetration Test values.

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