Nonlinear pushover analysis for pile foundations

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ABSTRACT

A technique frequently used by structural engineers for indicating ultimate limit state mechanisms in complex structures is known as pushover analysis. This is done with structural analysis software which is able to model not only elastic behaviour of structural elements but also nonlinear stiffness and failure of the elements. The purpose of this paper is to draw attention to some useful sets of equations which can represent the nonlinear lateral load behaviour of piles embedded in gravels, sands, and clays. The particular example presented is the pushover analysis of a wharf structure having vertical piles embedded in sandy gravel.

1 INTRODUCTION

The lateral load behaviour of piles is usually nonlinear. The most common way of handling this is to use computer software based on the Winkler spring idealisation of the localised interaction between the pile shaft and the soil; two examples being LPILE (Reese and van Impe (2001)) and RUAUMOKO (Carr (2005)). The intention of this paper is to demonstrate the use of an alternative to such software, an approach that gives the user more control over what is happening and leads, perhaps, to more insight.

A number of simple equations are needed for the calculation example discussed below; these are set-out below, more details and further examples can be found in Pender (2007). The first of these enables us to estimate the lateral displacement and rotation of the pile shaft where it emerges from the ground. At this stage both the pile and the soil in which it is embedded are elastic. Using flexibility coefficients the ground-line pile shaft lateral displacement and rotation are given by:

$$u_{gl} = f_{uH} H_{gl} + f_{uM} M_{gl} \quad (1)$$

$$\theta_{gl} = f_{\theta H} H_{gl} + f_{\theta M} M_{gl} \quad (2)$$

where: H_{gl} is the horizontal load applied at the groundline,
M_{gl} is the moment applied at the groundline,
u_{gl} is the pile shaft groundline lateral displacement,
θ_{gl} is the pile shaft groundline rotation,
and $f_{uH}, f_{uM}, f_{\theta H}, f_{\theta M}$ are the pile shaft groundline flexibility coefficients.

Frequently the actions are applied some distance above the groundline and the pile shaft displacements are required at that position. In this case the cantilever deflections of the pile shaft need to be added to the groundline deflections. With these displacements included equations 1 and 2 become:

$$u_{top} = u_{gl} + \theta_{gl} L + \frac{HL^3}{3EI} + \frac{ML^2}{2EI} \quad (3)$$

$$\theta_{top} = \theta_{gl} + \frac{HL^2}{2EI} + \frac{ML}{EI} \quad (4)$$

where: u_{top} and θ_{top} are the lateral displacement and rotation at the top of the pile shaft,
H and M are the horizontal force and moment applied at the top of the pile shaft,
L is the length of the pile shaft projection above the groundline,
EI is the flexural property of the pile section.

What is now required are values for the groundline pile shaft flexibility coefficients. These are available for various types of soil profile, Pender (2007), but in this paper we will present only those for a soil profile having a linear increase in Young's modulus with depth from zero at the ground surface. Such a case is thought to represent a profile of sand and gravel for all but the smallest shear strains. The relevant equations for this soil profile model are given by Budhu and Davies (1987). The ratio of the Young's modulus of the soil at a depth of one pile diameter to that of the modulus for the pile shaft is the basic parameter in the flexibility coefficient equations:

$$E_s = m D \qquad K = \frac{E_p}{m D} \quad (5)$$

where: m is the rate of increase in Young's modulus with depth.

$$f_{uH} = \frac{3.2K^{-0.333}}{mD^2} \qquad f_{uM} = f_{\theta H} = \frac{5.0K^{-0.556}}{mD^3} \qquad f_{\theta M} = \frac{13.6K^{-0.778}}{mD^4} \quad (6)$$

The above flexibility coefficients assume that the piles are "long", that is the lower parts of the pile shafts experience no lateral deflection or rotation from groundline actions. It is known that the length of pile shaft over which lateral deformations occur is several pile shaft diameters, this is defined as the "active length" given by:

$$L_a = 1.3DK^{0.222} \quad (7)$$

The position and value of the maximum pile shaft moment are given by:

$$L_{M\max} = 0.41L_a \qquad M_{\max} = I_{MH} DH_{gl} \qquad I_{MH} = aK^b \qquad a = 0.6f \quad b = 0.17f^{-0.3} \text{ and } f = \frac{M_{gl}}{DH} \quad (8)$$

The above equations enable one to estimate lateral deflections and rotations of the pile shaft at the groundline when the soil and the pile behave elastically. However, it is well known that pile deformation under lateral loading is usually nonlinear. Budhu and Davies extended their approach to handle nonlinear soil-pile interaction. They achieve this taking the displacements and maximum pile shaft moment for the elastic case and multiplying them by coefficients to account for soil nonlinearity. Their equations are:

$$\begin{aligned} u_{ygl} &= I_{uy} u_{Egl} \\ \theta_{ygl} &= I_{\theta y} \theta_{Egl} \\ M_{My} &= I_{My} M_{ME} \end{aligned} \qquad (9)$$

where: I_{uy}, $I_{\theta y}$, and I_{My} are yield influence factors,
u_{Egl} is the elastic groundline pile shaft displacement from equation (1),
θ_{Egl} is the elastic groundline pile shaft rotation from equation (2),
M_{ME} is the maximum elastic pile shaft moment from equation (8).

For linear soil modulus distribution with depth, the modification equations are:

$$\begin{aligned} I_{uy} &= 1 + \frac{h - k^{0.35}}{6k^{0.65}} \\ I_{\theta y} &= 1 + \frac{h - k^{0.35}}{11k^{0.35}} \\ I_{My} &= 1 + \frac{h}{20k^{0.35}} \end{aligned} \qquad (10)$$

where:

$$h = \frac{H_{gl}}{K_p \gamma D^3}$$

$$k = \frac{K \exp(0.07(\phi - 30))}{1000}$$

where: ϕ is the angle of shearing resistance of soil in which the pile is embedded

and γ is the unit weight of the soil.

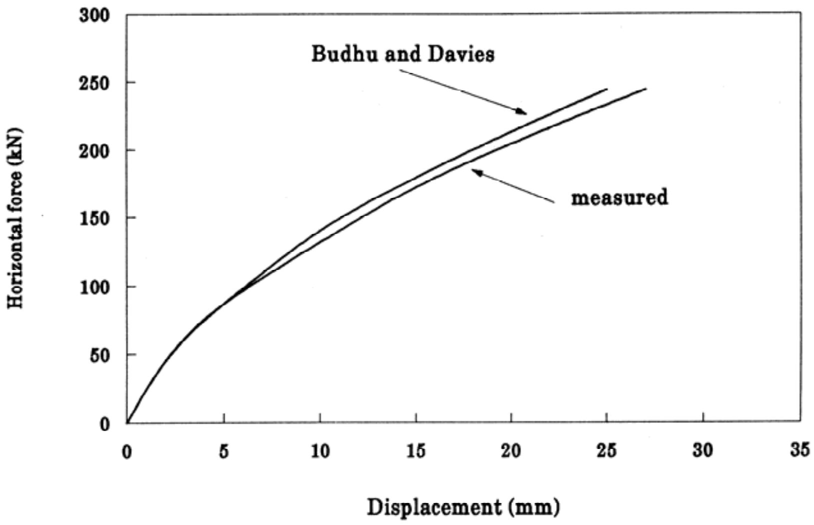
These equations have been found to predict the observed lateral load behaviour of laterally loaded piles in sand layers quite well, as shown in Fig. 1.

What we need is an expression for the nonlinear lateral deflection at the top of the pile shaft. There are two cases that need to be considered; the case of a free head pile where there is zero moment at the top of the pile shaft and the fixed head case where a moment is applied at the pile head to prevent rotation.

We now have all the tools required for the pushover analysis of the wharf structure.

2 PUSHOVER ANALYSIS OF A WHARF STRUCTURE

Results of calculations for the complete lateral load response of the wharf structure shown in Fig. 2 are presented in this section. The calculations were done using Mathcad (PTC 2007). As well as the ground properties we need the moment capacity of the reinforced concrete piles; this is taken from the NZ Concrete Society document, CCANZ (1995).



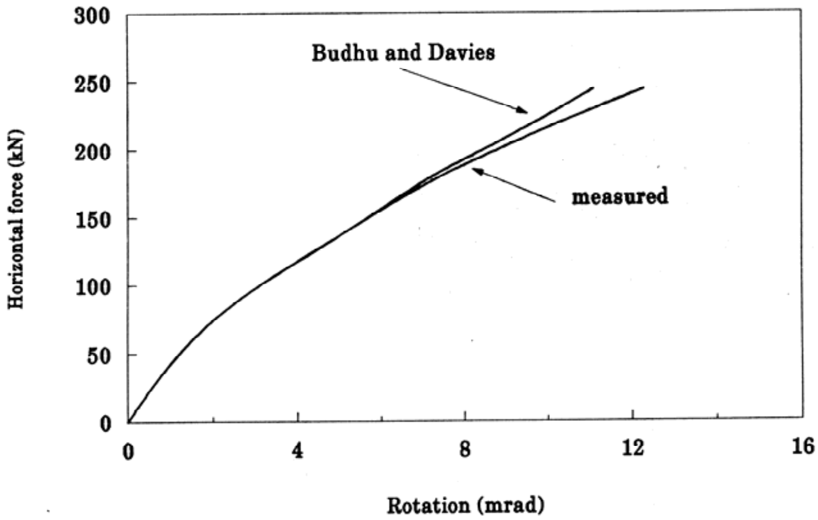


Figure 1: Groundline horizontal displacement and rotation for the Mustang Island pile test compared with the predictions of equations (9) and (10) (after Pranjoto (1992) & Reese et al (1974)).

As with any numerical modelling of this type a number of assumptions are made. The more important of these are:

- the piles are vertical and have constant section and properties over the active length,
- the wharf deck is rigid,
- the soil modulus increases linearly from zero at the ground surface (other models are possible),
- the effect of the sloping ground has little effect of the calculated stiffness (probably not a restrictive assumption when the linear increase of modulus with depth model is used),
- the piles are sufficiently far apart that the “shadowing” effect on the lateral load capacity of pile groups is not significant,
- the slope material is stable enough to sustain the pile lateral reaction forces,
- because of the nonlinear pile-soil behaviour there will be negligible interaction between the piles (could be significant if the modelling was limited to elastic behaviour of the soil),
- the moment capacity of the pile sections is estimated for zero axial load.

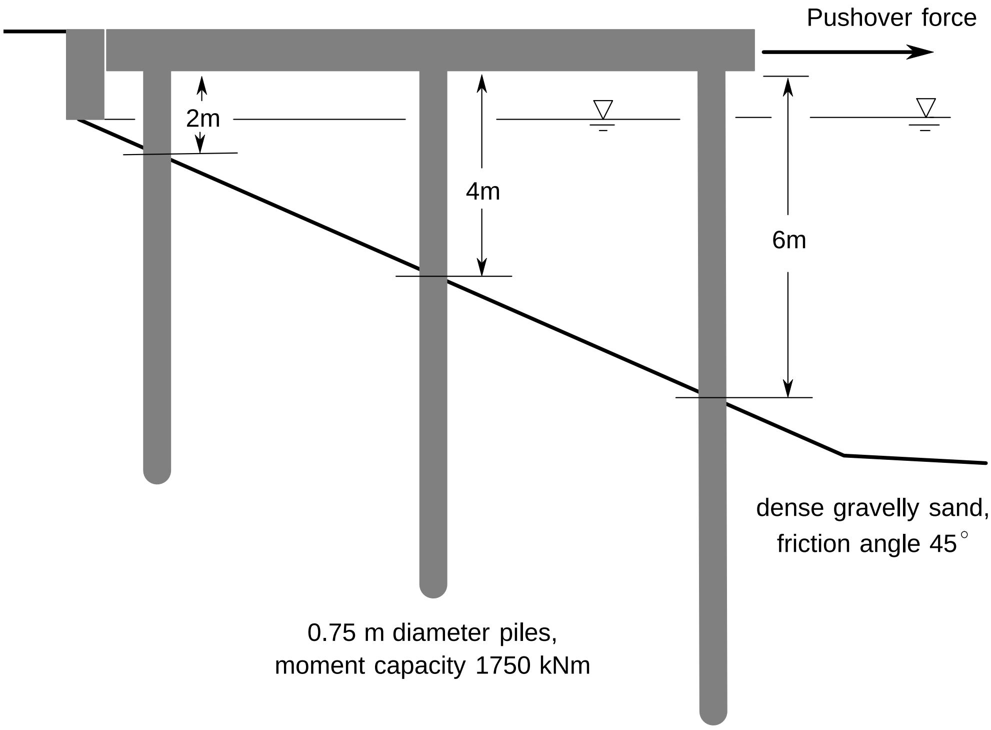


Figure 2: Wharf structure configuration

Figure 3 shows how the system responds to a gradually increasing lateral load when the connections between the pile shafts and the deck structure are pinned. What is first calculated is the lateral response of each pile, these curves are also shown in the Fig. 3. It can be seen that for each pile eventually the pile section yields and no further shear can be carried. Since the lateral displacements of the pile tops are all the same, because of the assumed rigidity of the deck, the global response is obtained by simple addition of the three curves at a series of common displacements. All the calculations for Fig. 3 were done using Mathcad, PTC (2007).

An obvious, and probably more realistic, extension of the work would be to have fixed connections between the piles and the wharf structure. This is easily accomplished by the method outlined above, but first one has to evaluate the fixing moments at the pile-deck connections. Space precludes illustrating this calculation here.

3 CONCLUSIONS

The paper demonstrates how relatively simple pile stiffness equations, for both linear and nonlinear soil-pile interaction, can be used to evaluate what is, at first look, quite a complex soil-structure interaction problem. For lateral pile-soil interaction it is very important to be able to take account of the nonlinear load deformation response of the pile. The Davies and Budhu equations provide such a convenient way of doing this, which deserves to be used more often as it has the potential to be applied to many more cases than the example discussed herein.

An underlying motivation of the paper is to provide a simple tool that can be used in place of pre-packaged software, or to provide the user with an independent way of checking that the software is working correctly.

Finally, a useful feature of this paper is that it illustrates the effectiveness of Mathcad in performing quite complex calculations in a manner that makes the calculation steps transparently clear.

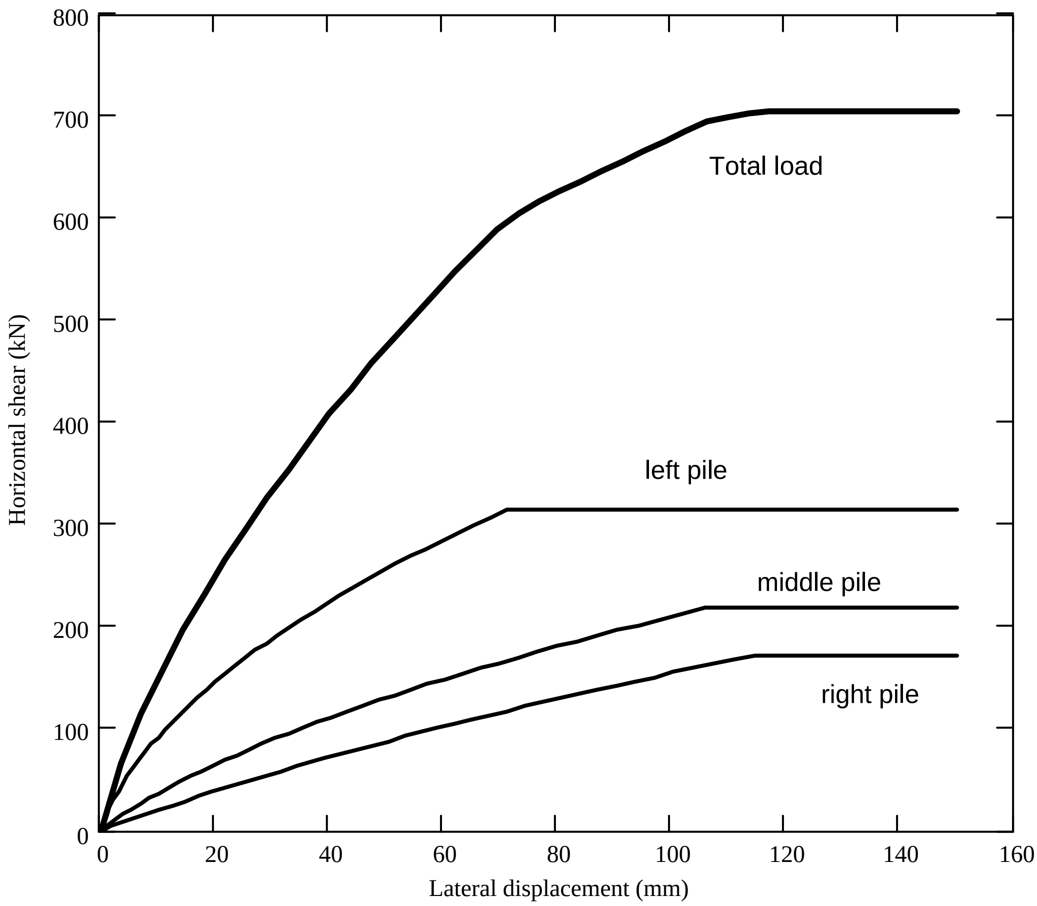


Figure 3: Push-over analysis of the wharf structure, pinned pile-wharf connections.

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